

Completing the Square: Lesson Plan

Overview:

Students will learn about the rich history of Islamic Mathematics, and will gain an understanding of the relationship of Islamic mathematics to modern mathematics. This unit requires some basic knowledge of algebra, and so is geared towards high school students or middle-school students learning algebra. Students will learn some of the methods used by Muslim mathematicians in “completing the square.” Using the history intertwined with the mathematics lesson, students will be more interested in the lesson and have a better understanding of completing the square.

Objectives:

Students will be able to:

1. Recount a brief history of Islamic mathematics and the expansion of Greek mathematics in the Arab world
2. Solve for x by completing the square

Activity:

Opening / Hook:

1. Students should be familiar with algebraic methods for solving quadratic equations, such as factoring. First, review these methods, with examples such as the following:

$$\begin{array}{ll} x^2 + 2x - 8 = 0 & x^2 + 10x + 25 = 0 \\ (x + 4)(x - 2) = 0 & (x + 5)^2 = 0 \\ x = -4, 2 & x = -5 \end{array}$$

2. Now, present the equation $x^2 - 2x - 6 = 0$. Ask the students how to solve an equation like this for a positive root, since it does not factor.

Introduce New Material:

1. Introduce the concept of “completing the square.” The goal of completing the square is to manipulate an equation into one that factors nicely, like the examples above. Begin with an easier example ($x^2 + 10x - 39 = 0$), one that *can* be factored nicely, so that students can solve the problems both ways, to see that they get the same answer.
2. First, give students a brief history of the “completing the square method, which was formulated by al-Khwārizmī when algebra was invented (information found on pages 1-2 of the “Islamic Mathematics” information packet). Then, follow the steps from pages 3-5 of the “Islamic Mathematics” information packet. These teach students to complete the square using the method of al-Khwārizmī. It might be easiest for students to use the second method (pp 4-5), but both should be presented.
3. Show the solution to the example problem ($x^2 + 10x - 39 = 0$) by factoring so that students can recognize that factoring and completing the square methods both give the same solution.

Guided Practice:

1. Pass out the Completing the Square worksheet, and help students to complete the first problem ($x^2 - 2x - 6 = 0$) by completing it on the board.

Independent Practice:

1. Have students work in groups of 2-3 people to complete the rest of the worksheet. (Make sure the students are keeping their plus and minuses the same during completing the square, depending on the sign in the original problem!) Walk around the room to help students who have trouble completing any of the problems.

Closing / Assessment:

1. For homework, assign problems similar to these (from a textbook or worksheet), asking students to solve the problems either by factoring or by completing the square. Or, have students complete side 1 of the worksheet in class and side 2 for homework.

Teachers: Please contact Angela Williams, CSAMES Outreach Coordinator, at aswillms@illinois.edu for the answer key to the worksheet.